Statistics Equation Sheet Midterm II

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Table 1: Lecture 4-5: One-Way Analysis of Variance (ANOVA) - Source Table Equations

Source	SS	*df*	MS	F
Between group variance - A	$[A]-[T]$	$(a-1)$	$\frac{SS_A}{df_A}$	$\frac{MS_A}{MS_{S/A}}$
Within group variance - S/A	$[Y]-[A]$	$a(n-1)$	$\frac{SS_{S/A}}{df_{S/A}}$	
Total	$[Y]-[T]$	$an-1$		

Table 2: Lecture 4-5: One-Way Analysis of Variance (ANOVA) - Bracket Terms

Variable	Sums	Description
$[A]$	$\frac{\sum A_j^2}{n}$	Square the sum of every group and together and divide by the number in each groups
$[Y]$	$\sum Y_{ij}^2$	Add up each squared score
$[T]$	$\frac{T^2}{an}$	Add each score together and square that number then divide by the total number of subjects

Table 3: Lecture 4-5: One-Way Analysis of Variance (ANOVA) - Calculating F in 6 easy steps

Step 1:	Add Every score in every group. Sum totals together to get T
Step 2:	Square every group sum. Then add group sums $\sum A_j^2$
Step 3:	Add every squared score in every group. Then add sum $\sum Y_{ij}^2$
Step 4:	Use results to calculate the bracket terms.
Step 5:	Use bracket terms to calculate Sums of Squares for each variance component.
Step 6:	Place Sums of Squares in source table and complete source table to calculate F ratio.

Table 4: Planned Comparisons

Variable	Equations	Definition
$\hat{\psi}$	$\sum c_j \bar{Y}_j$	Observed value of the contrasts
SS_ψ	$\frac{n\hat{\psi}^2}{\sum c_j^2}$	Sum of Squares
SS_ψ	$\frac{\hat{\psi}^2}{\sum \left(c_j^2/n_j\right)}$	Sum of Squares Unequal Sample Sizes
F	$\frac{MS_\psi}{MS_{S/A}}$	Significance of contrast
df	1	Degrees of Freedom

Table 5: Lecture 8:Two-way ANOVA

Term	Equation	Definition
Familywise Error	$\alpha_{FW} = 1 - (1-\alpha)^c$	The error rate for a family of comparisons
Bonferroni	$\alpha = \frac{\alpha_{FW}}{c}$	Technique for controlling α_{FW} error.
c	$\frac{\alpha(\alpha-1)}{2}$	Number of pair wise comparisons
Tukey's HSD	$HSD = q_a \left(\sqrt{\frac{MS_{S/A}}{n}}\right)$	Comparison of a critical value to the difference between sample means. The n's should be equal
Scheffe's	$F_{Scheffe} = (a-1)\, F_{omnibus}$	The most conservative of all post hoc test but also the easiest. Use $F_{scheffe}$ as the critical value for all post hoc tests.
Planned Comparisons		Is a theoretically motivated mean differences that you include in your experimental design.
Familywise Comparisons		Are families of comparisons and they, are only somewhat planned
Experimentwise Comparisons		Post Hoc and include every possible comparison that could be made from the data. They are a superfamily that inlcude all possible families. Very stringent guidelines are set up to control for experimentwise error α_{EW}
Polynomial Comparison		
Helmert Comparison		Contrasts that are mutually orthogonal
Pairwise Comparison	$a\,(a-1)\,/2$	To compare each group with every other group. Equation is the total number of comparisons.
Factor		Independent Variable
Levels		Specific treatment conditions.
Main Effect		Group differences for each factor. Same as between group effects.
Simple Effects		Group difference of one factor at a particular level of the other factor.
Interaction Effects		When the simple effect of one factor changes over levels of other factors.

Table 6: Lecture 8: Three easy steps to Tukey's HSD

1. Look up q_a (Page 588: Kepple) Number of groups (a) is in the columns and the degrees of freedom in the denominator are in the rows.

2. Compute HSD by applying the formula $HSD = q_a \left(\sqrt{\frac{MS_{S/A}}{n}} \right)$

3. Compare each mean difference to the Honestly Significantly Different difference. You only have a significant difference if the group means are greater.

Table 7: Lecture 8: (Chapter 11) 2-Way ANOVA - Source Table

Source	SS	*df*	MS	F
A	$[A] - [T]$	$(a-1)$	$\frac{SS_A}{df_A}$	$\frac{MS_A}{MS_{S/AB}}$
B	$[B] - [T]$	$(b-1)$	$\frac{SS_B}{df_B}$	$\frac{MS_B}{MS_{S/AB}}$
$A \times B$	$[AB] - [A] - [B] + [T]$	$(a-1)(b-1)$	$\frac{SS_{A \times B}}{df_{A \times B}}$	$\frac{MS_{A \times B}}{MS_{S/AB}}$
S/AB	$[Y] - [AB]$	$ab(n-1)$	$\frac{SS_{S/AB}}{df_{S/AB}}$	
Total	$[Y] - [T]$	$abn - 1$		

Table 8: Lecture 8: Main & Interaction Effect

	a_1	a_2	
b_1	$a_1 b_1$	$a_2 b_1$	B_1
b_2	$a_1 b_2$	$a_2 b_2$	B_2
	A_1	A_2	

A Main Effect: $A_1 \neq A_2$

B Main Effect: $B_1 \neq B_2$

Interaction Effect: $a_1 b_1 - a_2 b_1 \neq a_1 b_2 - a_2 b_2$

Table 9: Lecture 9: (Chapter 11) 2-Way ANOVA Bracket Terms

Variable	Sums	Means	Description
$[A]$	$\frac{\sum A_j^2}{bn}$	$bn\sum \bar{Y}_{A_j}^2$	The sum of (all scores at every level of A square) and divided by bn. The sums you squared are marginal sums.
$[B]$	$\frac{\sum B_k^2}{an}$	$an\sum \bar{Y}_{B_k}^2$	The sum of (all scores at every level of B square) and divided by an. The sums you square are marginal sums
$[AB]$	$\frac{\sum (AB_{jk})^2}{n}$	$n\sum \bar{Y}_{jk}^2$	The sum of every group sum squared and divided by n. The sums you squared are the cell sums
$[Y]$	$\sum Y_{ijk}^2$	$\sum Y_{ijk}^2$	The sum of every individual squared score
$[T]$	$\frac{T^2}{abn}$	$abn\bar{Y}_T^2$	(Every score added together and that sum squared) divided by abn

Table 10: Lexture 9: (Chapter 11) First Steps in a 2-Way ANOVA

Variable	Equations
AB_{jk}	$\sum Y_{ijk}$
$\sum Y^2$	
$\bar{Y}_{jk}$	$\frac{AB_{jk}}{n}$
s_{jk}	$\sqrt{\frac{\sum Y_{ijk}^2-(AB_{jk})^2/n}{n-1}}$
$s_{M_{jk}}$	$\frac{s_{jk}}{\sqrt{n}}$

Table 11: Lecture 9: (Chapter 11) Confidence Intervals for 2-Way ANOVA

Type of Confidence Interval	Equation	Estimated Standard Error
Population Mean μ_{jk}	$\bar{Y}_{jk} - ts_{M_{jk}} \leq \mu_{jk} \leq \bar{Y}_{jk} + ts_{M_{jk}}$	$s_{M_{jk}} = \sqrt{MS_{S/AB}/n_{jk}}$
Population Marginal μ_{Aj}	$\bar{Y}_{Aj} - ts_{M_A} \leq \mu_{Aj} \leq \bar{Y}_{Aj} + ts_{M_A}$	$s_{M_A} = \sqrt{MS_{S/AB}/bn}$

t = a value obtained from Appendix A.2 with $df = n_{jk} - 1$ and $df_{S/AB}$

Table 12: Lecture 9: (Chapter 11) Effect Size

Value	Variable	Equation
Complete Omega Squared	ω_{effect}^2	$\frac{\sigma_{effect}^2}{\sigma_{total}^2}$ $\frac{df_{effect}(F_{effect}-1)}{df_A(F_A-1)+df_B(F_B-1)+df_{A\times B}(F_{A\times B}-1)+abn}$
Partial Omega Squared	$\omega_{\langle effect\rangle}^2$	$\frac{\sigma_{effect}^2}{\sigma_{effect}^2+\sigma_{error}^2}$ $\frac{df_{effect}(F_{effect}-1)}{df_{effect}(F_{effect}-1)+abn}$

Table 13: Ch. 12: Comparisons for the Marginal Means

Variable	Equation
SS_{ψ_A}	$\frac{bn\hat{\psi}_A^2}{\sum {c^2}_j}$
$\hat{\psi}_A$	$\sum c_j \bar{Y}_{A_j}$
F_{ψ_A}	$\frac{MS_{\psi_A}}{MS_{S/AB}}$

Table 14: Ch. 12: Testing the simple effects - Computational Formulas

Variable	Equation
SS_A	$bn \sum \left(\bar{Y}_{A_j} - \bar{Y}_T\right)^2$
$SS_{A\ at\ b_k}$	$n \sum_j \left(\bar{Y}_{jk} - \bar{Y}_{B_k}\right)^2$
$SS_{A\ at\ b_k}$	$[A\ at\ b_k] - [T\ at\ b_k]$
$[A\ at\ b_k]$	$n \sum_j \bar{Y}_{jk}^2 = \frac{\sum_j (AB_{jk})^2}{n}$
$[T\ at\ b_k]$	$an\bar{Y}_{B_k}^2 = \frac{B_k^2}{an}$
$df_{A\ at\ b_k}$	$a - 1$

Table 15: Ch. 12: Partitioning of the Sums of Squares

$SS_{between} = SS_A + SS_B + SS_{AxB}$
$SS_{between} = \sum SS_{A\ at\ b_k} + SS_B$
$SS_{between} = SS_A + \sum SS_{B\ at\ A_j}$

Table 16: Ch.12: Computational Formulas

Variable	Equation
$\hat{\psi}_{A\ at\ b_k}$	$\sum_j c_j \bar{Y}_{jk}$
$SS_{\psi_A at\ b_k}$	$\frac{n\hat{\psi}^2_{A\ at\ b_k}}{\sum c_j^2}$

Table 17: Ch.12: Effect Sizes and Power for Simple Effects

Specialized Effect	Variable	Equation
main comparisons:	$\hat{\omega}^2_{\langle\psi_A\rangle}$	$\frac{F_{\psi_A}-1}{\left(F_{\psi_A}-1\right)+2bn}$
simple effect:	$\hat{\omega}^2_{\langle A\ at\ b_k\rangle}$	$\frac{df_{A\ at\ b_k}\left(F_{A\ at\ b_k}-1\right)}{df_{A\ at\ b_k}\left(F_{A\ at\ b_k}-1\right)+an}$
simple comparison:	$\hat{\omega}^2_{\langle\psi_A at\ b_k\rangle}$	$\frac{F_{\psi_A at\ b_k}-1}{\left(F_{\psi_A at\ b_k}-1\right)+2n}$

Table 18: Ch. 16: Terminology and definitions

Word	Definition
Blocking	Makes groups more homogeneous reducing group variance. Included in linear model effects not usually of much interest. Often random factors.
Random Factors	Where levels of the variable are randomly selected. Random error inflates the F ratio of the effect crossed with the random factor.
Blocking w/out Replication	One value per cell. No within group variance
RULE	If a factors is crossed with a random factor, the appropriate error term is the mean squared of that interaction.

Table 19: Ch. 16: Notation system for an $A \times S$ design

	Levels of Factor A				
Subjects	a_1	a_2	a_3	a_4	Sum
s_1	Y_{11}	Y_{12}	Y_{13}	Y_{14}	S_1
s_2	Y_{21}	Y_{22}	Y_{23}	Y_{24}	S_2
s_3	Y_{31}	Y_{32}	Y_{33}	Y_{34}	S_3
Sum	A_1	A_2	A_3	A_4	T

$$\bar{Y}_j = A_j/n$$
$$\bar{Y}_{S_i} = S_i/a$$
$$\bar{Y}_T = T/an$$

Table 20: Ch. 16: Computational formulas for the $A \times S$ design

Source	SS	*df*	MS	F
A	$[A]-[T]$	$(a-1)$	$\frac{SS_A}{df_A}$	$\frac{MS_A}{MS_{A\times S}}$
S	$[S]-[T]$	$(n-1)$	$\frac{SS_S}{df_S}$	
$A \times S$	$[Y]-[A]-[S]+[T]$	$(a-1)(n-1)$	$\frac{SS_{A\times S}}{df_{A\times S}}$	
Total	$[Y]-[T]$	$an-1$		

Table 21: Ch. 16: Bracket Terms

Variable	Sums	Means
$[A]$	$\frac{\sum A_j^2}{n}$	$n\sum \bar{Y}_A^2$
$[S]$	$\frac{\sum S_i^2}{a}$	$a\sum \bar{Y}_S^2$
$[Y]$	$\sum Y_{ij}^2$	$\sum Y_{ij}^2$
$[T]$	$\frac{T^2}{an}$	$an\bar{Y}_T^2$

Table 22: Ch. 16: Confidence Intervals for Single-factor within-subject design

Type of Confidence Interval	Equation	Estimated Standard Error
Population Marginal μ_j	$\bar{Y}_{Aj} - ts_{M_j} \leq \mu_j \leq \bar{Y}_{Aj} + ts_{M_j}$	$s_{M_1} = \frac{s_1}{\sqrt{n}}$
		$s_1 = \sqrt{\frac{\sum Y_{i1}^2 - (A_1)^2/n}{n-1}}$

t = a value obtained from Appendix A.2 with $df = n_{jk} - 1$ and $df_{S/AB}$

Table 23: Ch.16: Subject Contrasts and Error Variability

Variable	Equation
ψ_A	$\sum c_j \mu_j$ where $\sum c_j = 0$
SS_{ψ_A}	$\frac{n\hat{\psi}_A^2}{\sum c_j^2}$
F_{ψ_A}	$\frac{MS_{\psi_A}}{MS_{\psi_{A \times S}}}$

Table 24: Ch.16: Effect Size and Power

Variable	Equation
$\hat{\omega}_{\langle A \rangle}$	$\frac{(a-1)(F_A-1)}{(a-1)(F_A-1)+an}$
$R^2_{\langle A \rangle}$ or $\eta^2_{\langle A \rangle}$	$\frac{SS_A}{SS_A + SS_{A \times S}}$

Table 25: Ch.16: Tukey's HSD & Confidence Intervals

Variable	Equation
HSD	$q_a \left(\sqrt{\frac{MS_{Error\ Term}}{n}} \right)$
CI	$\left(\sqrt{\frac{MS_{Error\ Term}}{n}} \right) t_{crit} \pm \bar{Y}$